Lecture 17: Joins
Graduate School Information Panel

Thursday, Nov 9 @ 3:00PM
1240CS

Should I attend graduate school in CS?

How do I choose the right graduate program?

How do I prepare a competitive application?

Join us for a live Q&A with CS faculty, graduate students, and a graduate school admissions coordinator!
Lecture 17: Joins
Today’s Lecture

1. Recap: Select, Project

2. Joins

3. Joins and Buffer Management
1. Recap
Logical Plan = How

```
SELECT P.buyer
FROM Purchase P, Person Q
WHERE P.buyer = Q.name
AND Q.city = 'Madison'
```
Physical Plan = What

SELECT P.buyer
FROM Purchase P, Person Q
WHERE P.buyer = Q.name
AND Q.city = 'Madison'
Select Operator

**access path** = way to retrieve tuples from a table

- **File Scan**
  - scan the entire file
  - I/O cost: $O(N)$, where $N = \#\text{pages}$

- **Index Scan**:  
  - use an index available on some predicate  
  - I/O cost: it varies depending on the index
Index Matching

• We say that an index *matches* a selection predicate if the index can be used to evaluate it

• Consider a conjunction-only selection. An index matches (part of) a predicate if
  • Hash: only equality operation & the predicate includes *all* index attributes
  • B+ Tree: the attributes are a prefix of the search key (any ops are possible)
Choosing the Right Index

• **Selectivity** of an access path = *fraction* of data pages that need to be retrieved

• We want to choose the *most selective* path!

• Estimating the selectivity of an access path is a hard problem
Projection

**Simple case:** `SELECT R.a, R.d`
- scan the file and for each tuple output R.a, R.d

**Hard case:** `SELECT DISTINCT R.a, R.d`
- project out the attributes
- eliminate *duplicate tuples* (this is the difficult part!)
Projection: Sort-based

We can improve upon the naïve algorithm by modifying the sorting algorithm:

1. In Pass 0 of sorting, project out the attributes
2. In subsequent passes, eliminate the duplicates while merging the runs
Projection: Hash-based

2-phase algorithm:

• **partitioning**
  • project out attributes and split the input into B-1 partitions using a hash function $h$

• **duplicate elimination**
  • read each partition into memory and use an in-memory hash table (with a *different* hash function) to remove duplicates
2. Joins
What you will learn about in this section

1. RECAP: Joins

2. Nested Loop Join (NLJ)

3. Block Nested Loop Join (BNLJ)

4. Index Nested Loop Join (INLJ)
1. Nested Loop Joins
What you will learn about in this section

1. RECAP: Joins

2. Nested Loop Join (NLJ)

3. Block Nested Loop Join (BNLJ)

4. Index Nested Loop Join (INLJ)
RECAP: Joins
Joins: Example

\[ R \bowtie S \]

**Example:** Returns all pairs of tuples \( r \in R, s \in S \) such that \( r.A = s.A \)

\[
\begin{align*}
R &\quad S \\
A &| B & C &| A & D \\
1 &| 0 & 1 &| 3 & 7 \\
2 &| 3 & 4 &| 2 & 2 \\
2 &| 5 & 2 &| 2 & 3 \\
3 &| 1 & 1 &| & \\
\end{align*}
\]
Joins: Example

\[ R \bowtie S \]

**SELECT**  \( R.A, B, C, D \)
**FROM**  \( R, S \)
**WHERE**  \( R.A = S.A \)

*Example:* Returns all pairs of tuples \( r \in R, s \in S \) such that \( r.A = s.A \)

<table>
<thead>
<tr>
<th>R</th>
<th>S</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
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<tr>
<td>2</td>
<td>3</td>
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<tr>
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</tbody>
</table>
**Joins: Example**

Let's consider a join operation between two relations `R` and `S`. The operation is denoted as `R \bowtie S`. The SQL query to perform this join is:

```sql
SELECT R.A, B, C, D
FROM R, S
WHERE R.A = S.A
```

**Example:**

This operation returns all pairs of tuples `r \in R, s \in S` such that `r.A = s.A`.

### Relations `R` and `S`:

<table>
<thead>
<tr>
<th>R</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
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<td>1</td>
<td>0</td>
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</table>

<table>
<thead>
<tr>
<th>S</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>7</td>
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<tr>
<td>2</td>
<td>3</td>
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</tbody>
</table>

The result of the join operation is:

<table>
<thead>
<tr>
<th>A</th>
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<th>C</th>
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<tbody>
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</table>
## Joins: Example

\[ R \bowtie S \]

**Example:** Returns all pairs of tuples \( r \in R, s \in S \) such that \( r.A = s.A \)

```
SELECT R.A, B, C, D
FROM R, S
WHERE R.A = S.A
```

### Example Table

**R**

<table>
<thead>
<tr>
<th></th>
<th>A</th>
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<th>C</th>
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**S**

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**R**

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Joins: Example

\[ R \bowtie S \]

**Example:** Returns all pairs of tuples \( r \in R, s \in S \) such that \( r.A = s.A \)

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\begin{align*}
\text{SELECT} & \quad R.A, B, C, D \\
\text{FROM} & \quad R, S \\
\text{WHERE} & \quad R.A = S.A
\end{align*}
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| 2   | 3 | 4 | 3 |
| 2   | 5 | 2 | 2 |
| 2   | 5 | 2 | 3 |
| 3   | 1 | 1 | 7 |
Semantically: A Subset of the Cross Product

\[ R \bowtie S \]

**SELECT**  \( R.A, B, C, D \)
**FROM**  \( R, S \)
**WHERE**  \( R.A = S.A \)

**Example:** Returns all pairs of tuples \( r \in R, s \in S \) such that \( r.A = s.A \)

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Can we actually implement a join in this way?
Notes

• We write $R \bowtie S$ to mean join $R$ and $S$ by returning all tuple pairs where all shared attributes are equal

• We write $R \bowtie S$ on $A$ to mean join $R$ and $S$ by returning all tuple pairs where attribute(s) $A$ are equal

• For simplicity, we’ll consider joins on two tables and with equality constraints (“equijoins”)

However joins can merge > 2 tables, and some algorithms do support non-equality constraints!
Nested Loop Joins
Notes

• We are again considering “IO aware” algorithms: *care about disk IO*

• Given a relation R, let:
  • $T(R) = \# \text{ of tuples in } R$
  • $P(R) = \# \text{ of pages in } R$

• Note also that we omit ceilings in calculations... good exercise to put back in!
Nested Loop Join (NLJ)

Compute $R \bowtie S$ on $A$:

for $r$ in $R$:
    for $s$ in $S$:
        if $r[A] == s[A]$:
            yield $(r,s)$
Nested Loop Join (NLJ)

Compute $R \bowtie S$ on $A$:

```python
for r in R:
    for s in S:
        if r[A] == s[A]:
            yield (r, s)
```

Cost:

$P(R)$

1. Loop over the tuples in $R$

Note that our IO cost is based on the number of *pages* loaded, not the number of tuples!
Nested Loop Join (NLJ)

Compute $R \bowtie S$ on $A$:

for $r$ in $R$:
  for $s$ in $S$:
    if $r[A] = s[A]$:
      yield $(r, s)$

Cost:

$P(R) + T(R) \times P(S)$

1. Loop over the tuples in $R$
2. For every tuple in $R$, loop over all the tuples in $S$

Have to read all of $S$ from disk for every tuple in $R$!
Nested Loop Join (NLJ)

Compute $R \Join S$ on $A$:

1. Loop over the tuples in $R$.
2. For every tuple in $R$, loop over all the tuples in $S$.
3. Check against join conditions.

Note that NLJ can handle things other than equality constraints... just check in the if statement!

Cost:

$P(R) + T(R) \times P(S)$
Nested Loop Join (NLJ)

Compute $R \bowtie S$ on $A$:
- for $r$ in $R$:
  - for $s$ in $S$:
    - if $r[A] == s[A]$:
      - yield $(r, s)$

What would \textit{OUT} be if our join condition is trivial (if \textit{TRUE})? \textit{OUT} could be bigger than $P(R)*P(S)$... but usually not that bad

Cost:

$P(R) + T(R)*P(S) + \text{OUT}$

1. Loop over the tuples in $R$
2. For every tuple in $R$, loop over all the tuples in $S$
3. Check against join conditions
4. Write out (to page, then when page full, to disk)
Nested Loop Join (NLJ)

Compute $R \bowtie S$ on $A$:

For $r$ in $R$:
  For $s$ in $S$:
    If $r[A] == s[A]$:
      Yield $(r, s)$

Cost:

$$P(R) + T(R) \times P(S) + \text{OUT}$$

What if $R$ (“outer”) and $S$ (“inner”) switched?

$$P(S) + T(S) \times P(R) + \text{OUT}$$

Outer vs. inner selection makes a huge difference-
DBMS needs to know which relation is smaller!
IO-Aware Approach
Block Nested Loop Join (BNLJ)

Compute \( R \bowtie S \) on \( A \):

for each \( B-1 \) pages \( pr \) of \( R \):
  for page \( ps \) of \( S \):
    for each tuple \( r \) in \( pr \):
      for each tuple \( s \) in \( ps \):
        if \( r[A] = s[A] \):
          yield \((r, s)\)

Given \( B+1 \) pages of memory

Cost:
\[ P(R) \]

1. Load in \( B-1 \) pages of \( R \) at a time (leaving 1 page each free for \( S \) & output)

Note: There could be some speedup here due to the fact that we’re reading in multiple pages sequentially however we’ll ignore this here!
Compute $R \bowtie S$ on $A$:

for each $B-1$ pages $pr$ of $R$:

\text{for page ps of $S$:}

\text{for each tuple r in pr:}

\text{for each tuple s in ps:}

\text{if } r[A] == s[A]:

\text{yield } (r,s)

\vspace{1cm}

**Given $B+1$ pages of memory**

**Cost:**

$$P(R) + \frac{P(R)}{B-1} P(S)$$

1. Load in $B-1$ pages of $R$ at a time (leaving 1 page each free for $S$ & output)

2. For each $(B-1)$-page segment of $R$, load each page of $S$

\vspace{1cm}

**Note:** Faster to iterate over the \textit{smaller} relation first!
Block Nested Loop Join (BNLJ)

Compute \( R \bowtie S \) on \( A \):

for each \( B-1 \) pages \( pr \) of \( R \):
  for page \( ps \) of \( S \):
    for each tuple \( r \) in \( pr \):
      for each tuple \( s \) in \( ps \):
        if \( r[A] == s[A] \):
          yield \((r,s)\)

Given \( B+1 \) pages of memory

Cost:

\[
P(R) + \frac{P(R)}{B-1} P(S)
\]

1. Load in \( B-1 \) pages of \( R \) at a time (leaving 1 page each free for \( S \) & output)

2. For each \((B-1)\)-page segment of \( R \), load each page of \( S \)

3. Check against the join conditions

BNLJ can also handle non-equality constraints
Block Nested Loop Join (BNLJ)

**Compute** $R \bowtie S$ on $A$:

for each $B-1$ pages $pr$ of $R$:
  for page $ps$ of $S$:
    for each tuple $r$ in $pr$:
      for each tuple $s$ in $ps$:
        if $r[A] == s[A]$:
          yield $(r,s)$

Again, $OUT$ could be bigger than $P(R)*P(S)$... but usually not that bad

**Cost:**

$$P(R) + \frac{P(R)}{B-1} P(S) + OUT$$

1. Load in $B-1$ pages of $R$ at a time (leaving 1 page each free for $S$ & output)
2. For each $(B-1)$-page segment of $R$, load each page of $S$
3. Check against the join conditions
4. Write out

**Given** $B+1$ pages of memory
BNLJ vs. NLJ: Benefits of IO Aware

• In BNLJ, by loading larger chunks of R, we minimize the number of full *disk reads* of S
  • We only read all of S from disk for *every (B-1)-page segment of R*!
  • Still the full cross-product, but more done only *in memory*

\[
P(R) + T(R) \cdot P(S) + \text{OUT} \quad \rightarrow \quad P(R) + \frac{P(R)}{B-1} P(S) + \text{OUT}
\]

BNLJ is faster by roughly \(\frac{(B-1)T(R)}{P(R)}\)!
BNLJ vs. NLJ: Benefits of IO Aware

• Example:
  • R: 500 pages
  • S: 1000 pages
  • 100 tuples / page
  • We have 12 pages of memory (B = 11)

• NLJ: Cost = 500 + 50,000*1000 = 50 Million IOs ~= 140 hours

• BNLJ: Cost = 500 + \(\frac{500*1000}{10}\) = 50 Thousand IOs ~= 0.14 hours

A very real difference from a small change in the algorithm!
Smarter than Cross-Products
Smarter than Cross-Products: From Quadratic to Nearly Linear

• All joins that compute the full cross-product have some quadratic term
  • For example we saw:

\[
P(R) + \frac{P(R)}{B-1} P(S) + OUT
\]

• Now we’ll see some (nearly) linear joins:
  • \(\sim O(P(R) + P(S) + OUT)\), where again \(OUT\) could be quadratic but is usually better

We get this gain by taking advantage of structure- moving to equality constraints (“equijoin”) only!
Index Nested Loop Join (INLJ)

Compute $R \bowtie S$ on $A$:
Given index $idx$ on $S.A$:
for $r$ in $R$:
    $s$ in $idx(r[A])$:
    yield $r, s$

Cost:
$P(R) + T(R) * L + OUT$

where $L$ is the IO cost to access all the distinct values in the index; assuming these fit on one page, $L \sim 3$ is good est.

We can use an index (e.g. B+ Tree) to avoid doing the full cross-product!
3. Joins and Memory
What you will learn about in this section

1. Sort-Merge Join (SMJ)

2. Hash Join (HJ)

3. SMJ vs. HJ
Sort-Merge Join (SMJ)
What you will learn about in this section

1. Sort-Merge Join
2. “Backup” & Total Cost
3. Optimizations
Sort Merge Join (SMJ): Basic Procedure

To compute $R \bowtie S$ on $A$:

1. Sort $R$, $S$ on $A$ using *external merge sort*

2. *Scan* sorted files and “merge”

3. *[May need to “backup”- see next subsection]*

Note that if $R$, $S$ are already sorted on $A$, SMJ will be awesome!
SMJ Example: $R \bowtie S$ on $A$ with 3 page buffer

- For simplicity: Let each page be **one tuple**, and let the first value be $A$

We show the file HEAD, which is the next value to be read!
SMJ Example: $R \bowtie S$ on $A$ with 3 page buffer

1. Sort the relations $R$, $S$ on the join key (first value)
SMJ Example: \( R \bowtie S \) on \( A \) with 3 page buffer

2. Scan and “merge” on join key!
SMJ Example: \( R \bowtie S \) on \( A \) with 3 page buffer

2. Scan and “merge” on join key!
SMJ Example: $R \bowtie S$ on $A$ with 3 page buffer

2. Scan and “merge” on join key!

Disk

<table>
<thead>
<tr>
<th>R</th>
<th>S</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,a)</td>
<td>(0,j)</td>
<td>(0,a,j)</td>
</tr>
<tr>
<td>(3,j)</td>
<td>(3,g)</td>
<td></td>
</tr>
<tr>
<td>(5,b)</td>
<td>(7,f)</td>
<td></td>
</tr>
</tbody>
</table>

Main Memory

Buffer

(3,j,g)
SMJ Example: $R \bowtie S$ on $A$ with 3 page buffer

2. Done!

Disk

R

(0, a)  (3, j)  (5, b)

S

(0, j)  (3, g)  (7, f)

Output

(0, a, j)  (3, j, g)

Main Memory

Buffer
What happens with duplicate join keys?
Multiple tuples with Same Join Key: “Backup”

1. Start with sorted relations, and begin scan / merge...

Disk

Main Memory

Buffer

R

(0,a) (0,j) (0,b)

S

(0,j) (0,g) (7,f)

Output

(0,a) (0,j) (0,b)

(7,f) (0,g) (0,j)
Multiple tuples with Same Join Key: “Backup”

1. Start with sorted relations, and begin scan / merge...
Multiple tuples with Same Join Key: “Backup”

1. Start with sorted relations, and begin scan / merge...
Multiple tuples with Same Join Key: “Backup”

1. Start with sorted relations, and begin scan / merge...

Have to “backup” in the scan of S and read tuple we’ve already read!
Backup

• At best, no backup $\rightarrow$ scan takes $P(R) + P(S)$ reads
  • For ex: if no duplicate values in join attribute

• At worst (e.g. full backup each time), scan could take $P(R) \times P(S)$ reads!
  • For ex: if all duplicate values in join attribute, i.e. all tuples in R and S have the same value for the join attribute
  • Roughly: For each page of R, we’ll have to *back up* and read each page of S...

• Often not that bad however, plus we can:
  • Leave more data in buffer (for larger buffers)
  • Can “zig-zag” (see animation)
SMJ: Total cost

• Cost of SMJ is **cost of sorting** R and S...

• Plus the **cost of scanning**: \( \sim P(R) + P(S) \)
  - Because of backup: in worst case \( P(R) \times P(S) \); but this would be very unlikely

• Plus the **cost of writing out**: \( \sim P(R) + P(S) \) but in worst case \( T(R) \times T(S) \)

\[
\sim \text{Sort}(P(R)) + \text{Sort}(P(S)) + P(R) + P(S) + \text{OUT}
\]

Recall: \( \text{Sort}(N) \approx 2N \left( \left\lceil \log_B \frac{N}{2(B+1)} \right\rceil + 1 \right) \)

*Note: this is using repacking, where we estimate that we can create initial runs of length \( \sim 2(B+1) \)*
SMJ vs. BNLJ: Steel Cage Match

- If we have 100 buffer pages, P(R) = 1000 pages and P(S) = 500 pages:
  - Sort both in two passes: $2 \times 2 \times 1000 + 2 \times 2 \times 500 = 6,000$ IOs
  - Merge phase $1000 + 500 = 1,500$ IOs
  - $= 7,500$ IOs + OUT

What is BNLJ?

- $500 + 1000 \times \left\lfloor \frac{500}{98} \right\rfloor = 6,500$ IOs + OUT

- But, if we have 35 buffer pages?
  - Sort Merge has same behavior (still 2 passes)
  - BNLJ? 15,500 IOs + OUT!

SMJ is ~ linear vs. BNLJ is quadratic...
But it’s all about the memory.
A Simple Optimization: Merges Merged!

- SMJ is composed of a **sort phase** and a **merge phase**

- During the **sort phase**, run passes of external merge sort on R and S
  - Suppose at some point, R and S have \( \leq B \) (sorted) runs in total
  - We could do two merges (for each of R & S) at this point, complete the sort phase, and start the merge phase...
  - OR, we could combine them: do **one** B-way merge and complete the join!

*Given \( B+1 \) buffer pages*
Un-Optimized SMJ

Sort Phase
(Ext. Merge Sort)

- Split & sort
- Merge
- Merge

Merge / Join Phase

- Joined output file created!

Given \( B+1 \) buffer pages

Unsorted input relations

Lecture 17 > Backup
Simple SMJ Optimization

Sort Phase (Ext. Merge Sort)

- Split & sort
- Merge

<= B total runs

Split & sort
Merged with
B-Way Merge / Join

Given B+1 buffer pages

Unsorted input relations

Merge / Join Phase

Joined output file created!
Simple SMJ Optimization

- Now, on this last pass, we only do \( P(R) + P(S) \) IOs to complete the join!

- If we can initially split \( R \) and \( S \) into \( B \) total runs each of length approx. \( \leq 2(B+1) \), assuming repacking lets us create initial runs of \( \sim 2(B+1) \)- then we only need \( 3(P(R) + P(S)) + \text{OUT} \) for SMJ!
  - 2 R/W per page to sort runs in memory, 1 R per page to B-way merge / join!

- How much memory for this to happen?
  - \( \frac{P(R)+P(S)}{B} \leq 2(B + 1) \Rightarrow \sim P(R) + P(S) \leq 2B^2 
  - Thus, \( \max\{P(R), P(S)\} \leq B^2 \) is an approximate sufficient condition

If the larger of \( R,S \) has \( \leq B^2 \) pages, then SMJ costs \( 3(P(R)+P(S)) + \text{OUT} \)!
Takeaway points from SMJ

If input already sorted on join key, skip the sorts.
  - SMJ is basically linear.
  - Nasty but unlikely case: Many duplicate join keys.

SMJ needs to sort both relations
  - If max \{ P(R), P(S) \} < B^2 then cost is 3(P(R)+P(S)) + OUT
Hash Join (HJ)
What you will learn about in this section

1. Hash Join

2. Memory requirements
Recall: Hashing

• **Magic of hashing:**
  - A hash function $h_B$ maps into $[0,B-1]$
  - And maps nearly uniformly
  
• A hash **collision** is when $x \neq y$ but $h_B(x) = h_B(y)$
  - Note however that it will **never** occur that $x = y$ but $h_B(x) \neq h_B(y)$
  
• We hash on an attribute $A$, so our has function is $h_B(t)$ has the form $h_B(t.A)$.
  - **Collisions** may be more frequent.
Hash Join: High-level procedure

To compute $R \bowtie S$ on $A$:

1. **Partition Phase:** Using one (shared) hash function $h_B$, partition $R$ and $S$ into $B$ buckets

2. **Matching Phase:** Take pairs of buckets whose tuples have the same values for $h$, and join these
   1. Use BNLJ here; or hash again $\rightarrow$ either way, operating on small partitions so fast!

Note again that we are only considering equality constraints here.

We decompose the problem using $h_B$, then complete the join.
Hash Join: High-level procedure

1. **Partition Phase:** Using one (shared) hash function $h_B$, partition $R$ and $S$ into $B$ buckets

Disk

- $R$: $(0,a)$, $(0,a)$, $(3,j)$, $(0,j)$, $(3,b)$
- $S$: $(0,a)$, $(0,j)$, $(5,b)$

Disk

- $R_1$: $(0,a)$, $(0,j)$
- $R_2$: $(3,j)$, $(3,b)$
- $S_1$: $(0,a)$, $(0,j)$
- $S_2$: $(5,b)$

$h_B$

Note our new convention: pages each have two tuples (one per row)

More detail in a second...
Hash Join: High-level procedure

2. **Matching Phase:** Take pairs of buckets whose tuples have the same values for $h_B$, and join these.
Hash Join: High-level procedure

2. **Matching Phase:** Take pairs of buckets whose tuples have the same values for $h_B$, and join these.
**Hash Join Phase 1: Partitioning**

**Goal:** For each relation, partition relation into **buckets** such that if \( h_B(t.A) = h_B(t'.A) \) they are in the same bucket

Given \( B+1 \) buffer pages, we partition into \( B \) buckets:

- We use \( B \) buffer pages for output (one for each bucket), and 1 for input
  - The “dual” of sorting.
  - For each tuple \( t \) in input, copy to buffer page for \( h_B(t.A) \)
  - When page fills up, flush to disk.
How big are the resulting buckets?

• Given \( N \) input pages, we partition into \( B \) buckets:
  
  - Ideally our buckets are each of size \( \sim N/B \) pages

• What happens if there are hash collisions?
  
  - Buckets could be \( > N/B \)
  
  - We’ll do several passes...

• What happens if there are duplicate join keys?
  
  - Nothing we can do here... could have some skew in size of the buckets
How big do we want the resulting buckets?

- Ideally, our buckets would be of size $\leq B - 1$ pages
  - 1 for input page, 1 for output page, $B-1$ for each bucket

- Recall: If we want to join a bucket from R and one from S, we can do BNLJ in linear time if for one of them (wlog say R), $P(R) \leq B - 1$!
  - And more generally, being able to fit bucket in memory is advantageous

- We can keep partitioning buckets that are $> B-1$ pages, until they are $\leq B - 1$ pages
  - Using a new hash key which will split them...

Given $B+1$ buffer pages

Recall for BNLJ:
$$P(R) + \frac{P(R)P(S)}{B - 1}$$

We’ll call each of these a “pass” again...
Hash Join Phase 1: Partitioning

We partition into $B = 2$ buckets using hash function $h_2$ so that we can have one buffer page for each partition (and one for input).

For simplicity, we’ll look at partitioning one of the two relations - we just do the same for the other relation!

Recall: our goal will be to get $B = 2$ buckets of size $\leq B-1 \rightarrow 1$ page each.
Hash Join Phase 1: Partitioning

1. We read pages from R into the “input” page of the buffer...

Given $B+1 = 3$ buffer pages
Hash Join Phase 1: Partitioning

2. Then we use **hash function** \( h_2 \) to sort into the buckets, which each have one page in the buffer.
Hash Join Phase 1: Partitioning

2. Then we use hash function $h_2$ to sort into the buckets, which each have one page in the buffer.

Given $B+1 = 3$ buffer pages

- Input page 0
- Input page 1
- Output (bucket) pages

Diagram:
- Disk:
  - $(5, b)$
  - $(5, a)$
  - $(3, j)$
  - $(0, j)$
- Main Memory:
  - Buffer:
    - $(3, a)$
    - $(0, a)$
    - $(3, a)$
- $h_2(3) = 1$
Hash Join Phase 1: Partitioning

3. We repeat until the buffer bucket pages are full...

Given $B+1 = 3$ buffer pages
Hash Join Phase 1: Partitioning

3. We repeat until the buffer bucket pages are full...

Given $B+1 = 3$ buffer pages
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Given $B+1 = 3$ buffer pages
Hash Join Phase 1: Partitioning

3. We repeat until the buffer bucket pages are full... then flush to disk

Given $B+1 = 3$ buffer pages
Hash Join Phase 1: Partitioning

3. We repeat until the buffer bucket pages are full... then flush to disk
Hash Join Phase 1: Partitioning

Note that collisions can occur!

Given $B+1 = 3$ buffer pages

![Diagram showing partitioning and collisions in hash join phase 1]
Hash Join Phase 1: Partitioning

Finish this pass...

Given $B+1 = 3$ buffer pages
Hash Join Phase 1: Partitioning

Finish this pass...

Given $B+1 = 3$ buffer pages

![Diagram showing disk and main memory with partitioning and bucketing.]
Hash Join Phase 1: Partitioning

Finish this pass...

Given $B+1 = 3$ buffer pages

Collision!!!
Hash Join Phase 1: Partitioning

Finish this pass...

Given $B+1 = 3$ buffer pages
Hash Join Phase 1: Partitioning

Given $B + 1 = 3$ buffer pages

We wanted buckets of size $B - 1 = 1$...

However we got larger ones due to:

(1) Duplicate join keys

(2) Hash collisions
Hash Join Phase 1: Partitioning

To take care of larger buckets caused by (2) hash collisions, we can just do another pass!

What hash function should we use?

Do another pass with a different hash function, $h'_2$, ideally such that:

$h'_2(3) \neq h'_2(5)$

Given $B+1 = 3$ buffer pages
Hash Join Phase 1: Partitioning

Given $B+1 = 3$ buffer pages

Disk

B0
(0,a)
(0,j)
(0,j)

B1
(3,a)
(3,j)

B2
(5,a)
(5,b)

To take care of larger buckets caused by (2) hash collisions, we can just do another pass!

What hash function should we use?

Do another pass with a different hash function, $h'_2$, ideally such that:

$h'_2(3) \neq h'_2(5)$
Hash Join Phase 1: Partitioning

Given \( B+1 = 3 \) buffer pages

What about duplicate join keys? Unfortunately this is a problem... but usually not a huge one.

We call this unevenness in the bucket size skew
Now that we have partitioned R and S...
Hash Join Phase 2: Matching

- Now, we just join pairs of buckets from R and S that have the same hash value to complete the join!
Hash Join Phase 2: Matching

• Note that since \( x = y \rightarrow h(x) = h(y) \), we only need to consider pairs of buckets (one from R, one from S) that have the same hash function value.

• If our buckets are \( \sim B - 1 \) pages, can join each such pair using BNLJ in linear time; recall (with \( P(R) = B-1 \)):

\[
\text{BNLJ Cost: } P(R) + \frac{P(R)P(S)}{B-1} = P(R) + \frac{(B-1)P(S)}{B-1} = P(R) + P(S)
\]

Joining the pairs of buckets is linear! (As long as smaller bucket \( \leq B-1 \) pages)
Hash Join Phase 2: Matching

R \bowtie S on A
Hash Join Phase 2: Matching

To perform the join, we ideally just need to explore the dark blue regions = the tuples with same values of the join key A
Hash Join Phase 2: Matching

With a join algorithm like BNLJ that doesn’t take advantage of equijoin structure, we’d have to explore this whole grid!
Hash Join Phase 2: Matching

R \bowtie S on A

With HJ, we only explore the blue regions

= the tuples with same values of $h(A)$

We can apply BNLJ to each of these regions
Hash Join Phase 2: Matching

\[ R \bowtie S \text{ on } A \]

An alternative to applying BNLJ:

We could also hash again, and keep doing passes in memory to reduce further!
How much memory do we need for HJ?

• Given B+1 buffer pages

+ WLOG: Assume P(R) <= P(S)

• Suppose (reasonably) that we can partition into B buckets in 2 passes:
  • For R, we get B buckets of size \( \sim \frac{P(R)}{B} \)
  • To join these buckets in linear time, we need these buckets to fit in B-1 pages, so we have:

\[
B - 1 \geq \frac{P(R)}{B} \Rightarrow \sim B^2 \geq P(R)
\]
Hash Join Summary

• Given enough buffer pages as on previous slide...

  • **Partitioning** requires reading + writing each page of R,S
    • \( \rightarrow 2(P(R)+P(S)) \) IOs

  • **Matching** (with BNLJ) requires reading each page of R,S
    • \( \rightarrow P(R) + P(S) \) IOs

  • **Writing out results** could be as bad as \( P(R)\times P(S) \)... but probably closer to \( P(R)+P(S) \)

HJ takes \(~3(P(R)+P(S)) + OUT\) IOs!
SMJ vs. HJ
Sort-Merge v. Hash Join

- *Given enough memory*, both SMJ and HJ have performance:

\[
\sim 3(P(R)+P(S)) + OUT
\]

- "Enough" memory =

  - SMJ: \(B^2 > \max\{P(R), P(S)\}\)
  
  - HJ: \(B^2 > \min\{P(R), P(S)\}\)

Hash Join superior if relation sizes *differ greatly*. Why?
Further Comparisons of Hash and Sort Joins

• Hash Joins are highly parallelizable.

• Sort-Merge less sensitive to data skew and result is sorted
Summary

• Saw IO-aware join algorithms
  • Massive difference

• Memory sizes key in hash versus sort join
  • Hash Join = Little dog (depends on smaller relation)

• Skew is also a major factor